

Relativity in polarized electron scattering observables

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Abstract. Coincidence scattering of polarized electrons from nuclei with polarization transfer to outgoing nucleons is studied within the context of relativistic mean field theory. Effects introduced by the dynamical enhancement of the lower components of the bound and scattered nucleon wave functions are analyzed for the polarized response functions and transferred polarization asymmetries. Results obtained by projecting out the negative-energy components are compared with the fully-relativistic calculation. The crucial role played by the relativistic dynamics in some spin-dependent observables is clearly manifested even for low/medium values of the missing momentum. Kinematical relativistic effects are also analyzed. A discussion of the factorization approach and the mechanism for its breakdown is also briefly presented.

Quasielastic coincidence electron scattering reactions have provided over the years important insight into single-particle properties of nuclei. This is so because at quasielastic kinematics the reaction mechanism underlying $(e, e'N)$ reactions can be treated with confidence in the impulse approximation (IA), i.e., assuming the virtual photon attached to a single bound nucleon that absorbs the whole momentum (q) and energy (ω) (see [1, 2] for details).

A large fraction of the theoretical analyses of $(e, e'N)$ reactions in past years was carried out on the basis of non-relativistic calculations. Within this scheme, the bound and ejected nucleons are described by non-relativistic wave functions which are solutions of the Schrödinger equation with phenomenological potentials. Moreover, the current operator is also described by a non-relativistic

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expression derived directly from a Pauli reduction. Such non-relativistic reductions constitute the basis for the standard distorted-wave impulse approximation (DWIA) that has been widely used to describe $(e, e'N)$ experiments performed at intermediate energies [1, 2].

In the last decade some experiments performed have involved momenta and energies high enough to invalidate the non-relativistic expansions assumed in DWIA. A consistent description of these processes requires one to incorporate relativistic degrees of freedom wherever possible. An important effort has been made in recent years in order to incorporate relativity in the description of $(e, e'N)$ reactions. In particular, nuclear responses and cross sections have been investigated recently by our group using the relativistic mean field approach [3]. This constitutes the basis of the relativistic distorted-wave impulse approximation (RDWIA), where bound and scattered wave functions are described as Dirac solutions with scalar and vector potentials, and the relativistic free nucleon current operator is assumed.

Relativistic contributions can be cast into two general categories, kinematical and dynamical relativistic effects. The former are directly connected with the structure of the 4-vector current operator, compared with the non-relativistic one that usually involves p/M_N , q/M_N and ω/M_N expansions. The latter, dynamical relativistic effects, come from the difference between the relativistic and non-relativistic nucleon (bound and ejected) wave functions involved. Within these dynamical relativistic effects one may distinguish effects associated with the Darwin term (dynamical depression of the upper component of the scattered nucleon wave function in the nuclear interior that mainly affects the determination of spectroscopic factors at low missing momenta) and effects due to the dynamical enhancement of the lower components of the relativistic wave functions (expected to be especially relevant at high missing momenta, although they have proven to play an important role for some particular observables even at low/medium p values).

So far, fully-relativistic analyses of $(e, e'p)$ reactions have clearly improved the comparison with experimental data. In [3] it was shown that the non-local Darwin term causes an enhanced absorption when comparing RDWIA cross section with the DWIA one at moderate p values, thus predicting larger spectroscopic factors [3, 4]. For larger missing momenta the lower components of the relativistic wave functions start to play a more important role, enhancing the high momentum components of the nucleon wave functions.

In recent work we have undertaken a systematic study of the relativistic effects in different observables. We started with the relativistic plane-wave impulse approximation (RPWIA), i.e., neglecting final-state interactions (FSI) between the outgoing nucleon and the residual nucleus. Although being an oversimplification, the RPWIA approach allows one to simplify the analysis, disentangling the relativistic effects from other distortion effects. The presence of negative-energy components in the relativistic bound nucleon wave function was shown to be very important for some observables even at low/moderate values of the missing momentum. In particular, the interference TL and TT responses were proved to be very sensitive to dynamical effects of relativity

affecting the lower components. These results persist also in modeling where FSI are included.

Following similar arguments, we have presented in recent work [5] a systematic study of the new response functions that enter in the description of $A(e, e'N)B$ processes. Since spin and relativity go hand in hand, one may *a priori* consider the relativistic approach to be better suited to describe nucleon polarization observables. Kinematical and dynamical relativistic effects for responses and polarization observables have been analyzed in detail within the plane wave limit for the outgoing nucleon. In work in progress effects in the final state are also being incorporated through relativistic FSI. They are briefly summarized in this work.

In the case of $A(e, e'N)B$ reactions, the hadronic response functions are usually given by referring the recoil nucleon polarization to the coordinate system defined by the axes: \mathbf{l} (parallel to the momentum \mathbf{p}_N of the outgoing nucleon), \mathbf{n} (perpendicular to the plane containing \mathbf{p}_N and the momentum transfer \mathbf{q}), and \mathbf{s} (determined by $\mathbf{n} \times \mathbf{l}$). In terms of the polarization asymmetries, the differential cross section can be expressed in the form

$$\frac{d\sigma}{d\varepsilon' d\Omega_e d\Omega_N} = \frac{\sigma_0}{2} [1 + \mathbf{P} \cdot \boldsymbol{\sigma} + h(A + \mathbf{P}' \cdot \boldsymbol{\sigma})] , \quad (1)$$

where σ_0 is the unpolarized cross section, A denotes the electron analyzing power, and \mathbf{P} (\mathbf{P}') represents the induced (transferred) polarization. A general study of the properties and symmetries of all of these responses and polarizations can be found in [6]. In the plane wave limit for the outgoing nucleon, the induced polarization \mathbf{P} and the analyzing power A are zero. In terms of nuclear responses, from the total of eighteen response functions, only nine survive within RPWIA. Four, R_0^L , R_0^T , R_0^{TL} and R_0^{TT} , represent the unpolarized responses and the five remaining, $R_l^{T'}$, $R_s^{T'}$, $R_l^{TL'}$, $R_s^{TL'}$ and $R_n^{TL'}$ (this one enters only for out-of-plane kinematics), depend explicitly on the recoil nucleon polarization and only enter when the electron beam is also polarized. In this work, we restrict our discussion to these observables known as transferred polarization responses and/or transferred asymmetries.

First, we consider the case of the plane wave limit for the outgoing nucleon wave function (RPWIA). The role played by the negative-energy projection components in the polarized responses was discussed in [5]. There we show that in two responses, $R_l^{T'}$ and $R_s^{TL'}$, the contribution of the negative-energy projections is almost negligible, that is, dynamical relativistic effects from the bound nucleon wave function do not significantly affect these responses. On the contrary, $R_s^{T'}$ and $R_l^{TL'}$ are much more sensitive. This result resembles what appeared for the unpolarized interference TL response [7]. Hence there exists a strong discrepancy between RPWIA results and those corresponding to the standard factorized PWIA.

Dynamical enhancement of the lower components in the bound nucleon wave function is even more clearly seen when analyzing the transferred polarization asymmetries (Fig. 1). Here the fully-relativistic RPWIA results (dashed lines) corresponding to the Coulomb gauge with the $CC1$ and $CC2$ choices of the cur-

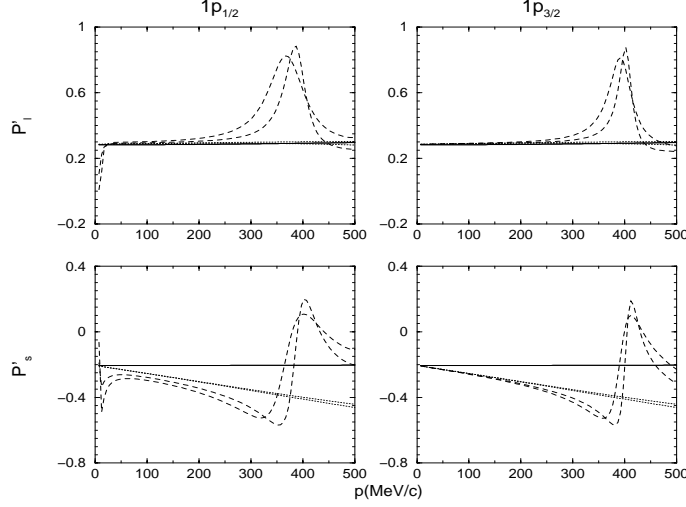


Figure 1. Transferred polarization asymmetries P'_l and P'_s in the plane wave limit for the outgoing nucleon. Fully-relativistic results (dashed lines) are compared with their positive-energy projection contributions (dotted lines). Thin lines correspond to the $CC1$ current operator and thick lines to $CC2$. We also show for comparison the static limit result (solid line).

rent (see ref. [5] for details on the current operators), are compared with the transferred polarizations obtained by projecting out the negative-energy components (dotted lines). Results for $p_{1/2}$ (left-hand panels) and $p_{3/2}$ (right-hand panels) are shown. Kinematics corresponds to $q = 1$ GeV/c, $\omega = 440$ MeV/c and forward ($\theta_e = 23^\circ$) scattering angle. First, note the difference between the relativistic and projected results observed at very small missing momentum values for the $p_{1/2}$ shell. This effect comes directly from the quantum number $\bar{\ell}$ involved in the lower component of the bound state wave function ($\bar{\ell} = 0$ for $p_{1/2}$). Moreover, it is also important to point out that fully-relativistic and positive-energy projected results typically do not differ appreciably for p -values up to ~ 300 MeV/c. For $p > 300$ MeV/c relativistic and projected results start to deviate from each other. This general behaviour is what one expects because of the clear dominance of the positive-energy projection component of the momentum distribution in the region $p \leq 300$ MeV/c [7]. On the contrary, for $p > 300$ MeV/c the negative-energy components are similar to or even larger than the positive ones, and hence the effects of the dynamical enhancement of the lower components in the bound relativistic wave functions are clearly visible in the transfer polarization asymmetries.

Obviously, final state interactions (FSI) between the ejected nucleon and the residual nucleus should be included in the analysis of $A(e, e'N)$ processes in order to compare with data. In fact, FSI introduce significant modifications in the responses and transferred polarization asymmetries. However, the high sensi-

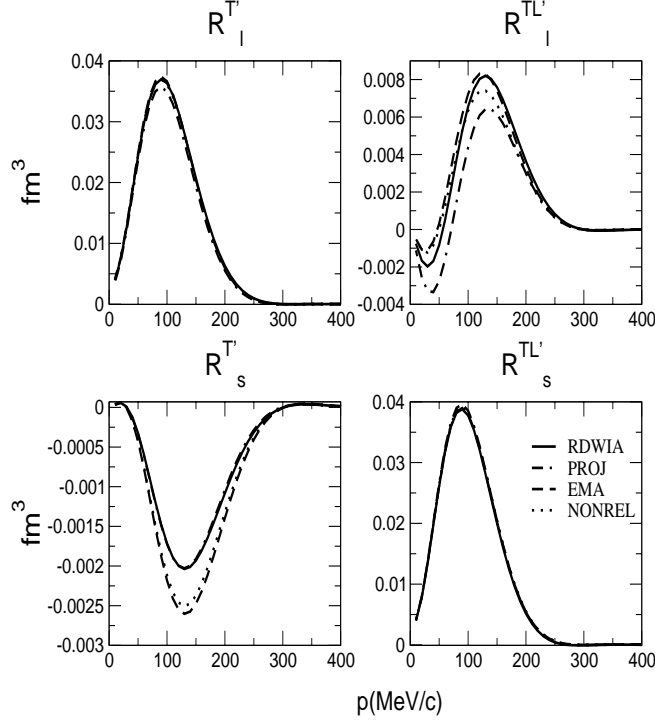


Figure 2. Polarized response functions for the $1p_{1/2}$ shell. Fully-relativistic response (solid line) is compared with the projected one (dot-dashed), EMA (dashed) and non-relativistic approach (dotted).

tivity of polarization-related-observables to negative-energy projections shown within RPWIA is maintained in the relativistic distorted-wave impulse approximation (RDWIA). This was already the case for the unpolarized interference longitudinal-transverse response R^{TL} and A_{TL} -asymmetry. The analysis presented in [8] proves the crucial role played by both kinematical and dynamical relativistic effects in order to fit the experiment. In particular, the richness shown by the structure of the left-right asymmetry (A_{TL}) is only consistent with predictions of relativistic calculations that include dynamic enhancement of the lower components of Dirac spinors.

Similar comments can be also applied to the transferred polarization observables. In particular, the different sensitivity to relativistic effects shown by the transferred polarized responses within RPWIA, persists when RDWIA calculations are involved. This is illustrated in Fig. 2 where we present the polarized responses corresponding to the $p_{1/2}$ -shell in ^{16}O . Kinematics has been selected as in the previous figure. We compare the fully distorted relativistic calculation using the Coulomb gauge and the current operator CC2 (solid line) with the results after projecting the bound and scattered proton wave

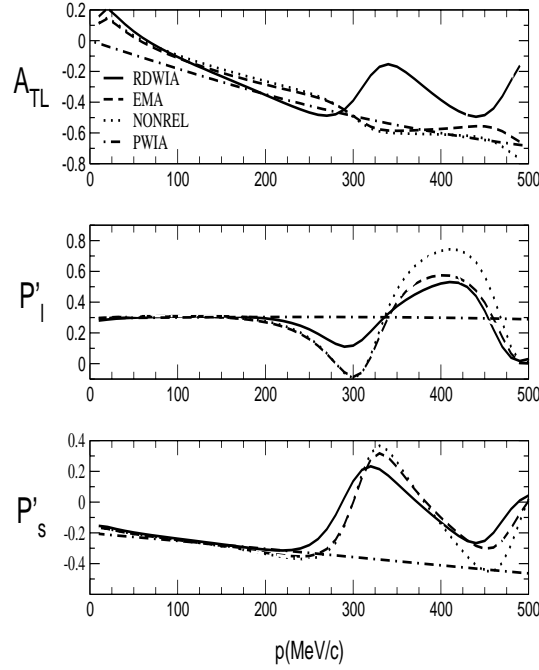


Figure 3. Asymmetry A_{TL} and transferred polarization ratios P'_l and P'_s . RDWIA results (solid) are compared with EMA (dashed), non-relativistic (dotted) and factorized PWIA (dot-dashed) calculations.

functions over positive-energy states (dot-dashed) and using the asymptotic momenta (dashed). This last approach is equivalent to the effective momentum approximation (EMA). Also for comparison we show a non-relativistic calculation where the non-relativistic current operator (obtained from a Pauli reduction) corresponds to the expression given in [9]. Note that two responses, $R_s^{TL'}$ and $R_l^{T'}$, are very insensitive to dynamical and kinematical relativistic effects. On the contrary, effects due to relativity are clearly visible for $R_l^{TL'}$ and $R_s^{T'}$. This result agrees with the findings within RPWIA [5].

Finally, in Fig. 3 we show the RDWIA results for the left-right asymmetry A_{TL} and transferred polarization asymmetries P'_l and P'_s . The curves are labelled as in the previous figure. It is particularly interesting the comparison between the fully relativistic results and the ones corresponding to the asymptotic projection (equivalent to EMA) and the non-relativistic calculation. Note that the richer structure shown by the relativistic calculation in A_{TL} is basically lost within the EMA and non-relativistic approaches. Moreover, these results get closer to the factorized PWIA calculation. This modest deviation from the factorized (free) result is mainly due to the spin-orbit coupling in the

final state. The behaviour shown by the transferred polarization asymmetries is clearly different. Here, the EMA and non-relativistic calculations do not change substantially the structure shown by the fully relativistic result, being very different to the factorized (free) one. This means that the spin-orbit coupling in the final wave function breaks down completely factorization in the polarized observables.

Summarizing, in this work we have analyzed outgoing nucleon polarized observables within a relativistic mean field approach. We have shown that dynamical relativistic effects, namely the enhancement of the lower components of Dirac wave functions, affect very significantly the observables. This was already the case for the unpolarized observables, R^{TL} and A_{TL} . However, here we also show that the behaviour presented by the transferred polarization asymmetries is clearly different from the one corresponding to the left-right asymmetry.

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